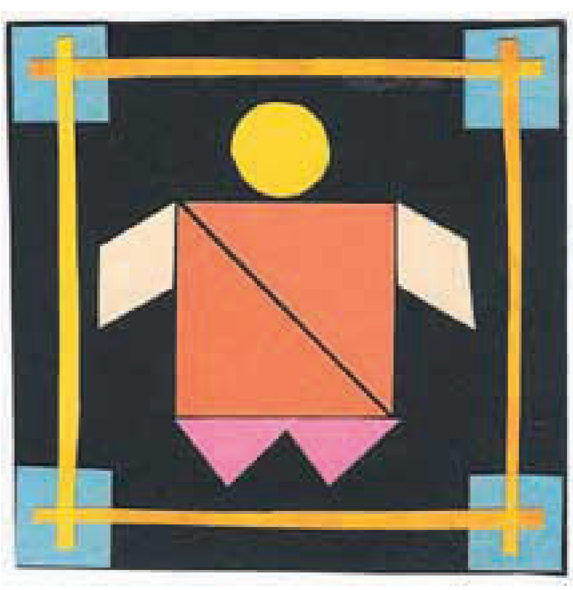
ALGEBRA in the Early Years? Yes!

Jennifer Taylor-Cox

What? Algebra? Don't say yikes! Say yes! It is never too early to start thinking in terms of algebra. Of course, I am not suggesting that we ask kindergartners to solve $3y - 6 = 45$. But we do need to offer young children a solid foundation of algebraic thinking. It is no longer satisfactory to simply "cover" patterns or "introduce" algebraic concepts in brief miniunits. We cannot be content with teaching only two-color patterns or offering math as a concept disconnected from the lives of children.

Activities that support algebraic thinking in the early years are addressed in the joint position statement issued by the National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM). It calls for the advancement of "good beginnings" with "high-quality, challenging, and accessible mathematics education" (NAEYC & NCTM 2002, 1). Algebra in the early years establishes the necessary groundwork for ongoing and future mathematics learning.



Why is algebra important?

Algebra in the early years establishes the necessary groundwork for ongoing and future mathematics learning.

Algebra is a generalization of the ideas of arithmetic where unknown values and variables can be found to solve problems. Algebra has long served both as a gate and a barrier for students (Lott 2000). High school students who take algebra typically proceed through the gate to higher education. Those who do not face obstacles in further academic pursuits, such as fulfilling noncredit course requirements or failing to be accepted into institutions of higher learning. Thus, algebra is a gatekeeper subject (Moses 2001). Fortunately, mathematics educators and policy makers have declared "algebra for all" as an initial step toward assuring equal educational opportunities.

Encouraging all students to take algebra is an important initial step, but the problem, say some educators, lies in the preparedness of students. As NCTM (2000) suggests, we can prepare students to be successful in algebra if we begin teaching them to think algebraically in the early years. "Algebra for all" must be preceded by "Algebra in the early years!"

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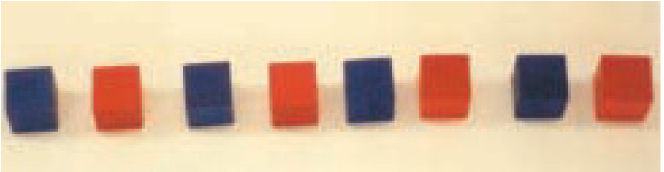


Figure 1: Block pattern

What does algebra in the early years look like?

When we map the early experiences needed for success in learning formal algebra, we find several interconnected concepts that are developmentally appropriate and very applicable to early childhood education. The central ideas promoted in the national algebra standard for young children are (1) patterns, (2) mathematical situations and structures, (3) models of quantitative relationships, and (4) change (NCTM 2000). All of these major concepts "enhance children's natural interest in mathematics and their disposition to use it to make sense of their physical and social worlds" (NAEYC & NCTM 2002, 5). Clearly, these "big ideas" need to be interconnected, fundamental aspects of an early childhood mathematics program.

Patterns

Patterns serve as the cornerstone of algebraic thinking. "Childen watch the sun setting every day; listen to stories, songs, and verses that follow patterns; notice how a puppy plays and sleeps on a schedule; jump rope to patterned chants; and skip over sidewalk bricks laid in patterns" (Copley 2000, 83). "Recognizing, describing, extending, and translating patterns" (NCTM 2000, 90) encourage children to think in terms of algebraic problem solving. Working with patterns invites young children to identify relationships and form generalizations (NCTM 2000). When we consider patterns, many of us focus on a series that repeats. For example, "day, night, day, night, day, night . . ." Indeed, the repeating pattern is one of the distinctive forms of patterning. As the name infers, repeating patterns contain a segment that continuously recurs. The segment can vary in size and level of complexity, but the simplest includes just two items.

With toddlers we can begin building the foundations of algebra through everyday experiences with patterns. For example, a teacher shares with two-year-olds a "clap, tap, clap, tap, clap, tap . . ." pattern. The children are invited to join in as soon as they are ready to do so. For toddlers, simple patterns involving movement and rhythm are quite beneficial.

Simple position patterns such as "up, down, up, down . . ." or "top, side, front, top, side, front . . ." encourage children to use spatial orientations as the repeating features.

In a prekindergarten classroom, four-year-old Ethan places blocks in a simple pattern (see Figure 1). Clearly, he knows something about repeating patterns. When Ms. Trep asks him to "read" the pattern, Ethan excitedly explains, "Well, it's blue, red, blue, red, blue, red, blue, red . . ." Ms. Trep encourages him to point to each block as he "reads" the pattern. Then, to help Ethan move to the next level, Ms. Trep sets up a series of three color blocks in a repeating format (green, yellow, blue, green, yellow, blue, green, yellow, blue . . .). She asks, "What comes next?" When Ethan says he's not sure, Ms. Trep asks him to read the pattern. In so doing, the child uses many senses; he points to the blocks, looks at them, says and hears the colors in the pattern. Ethan quickly realizes that the green block is next in the pattern. When Ms. Trep asks him to explain why it is a pattern, he replies, "You know it's a pattern because it goes again and again!"

In this situation, Ms. Trep helps Ethan assimilate information through his senses, allowing him to successfully tackle a more complex pattern. Ethan faces the developmentally appropriate degree of challenge offered to him within a comfortable and nurturing environment. After further similar experiences, he will be ready to try more complex patterns.

Another way to increase the level of pattern complexity is to use attributes other than color. Beginning with a series of two, a teacher can encourage children to focus on shape and size as repeating attributes. Patterns such as "triangle, circle, triangle, circle, triangle . . ." or "big, small, big, small, big, small . . ." invite children to continue repeating patterns using more complex attributes. It is important to remove color as an attribute when working on shape or size patterns. For example, "hexagon, trapezoid, hexagon, trapezoid . . ." may look like a shape pattern, but if all the hexagons are yellow and all the trapezoids are red, a child may simply focus on color. The attribute of focus is quite apparent when the child is asked to read the pattern. With shape or size patterns, teachers can negate color by providing all same-color shapes (for instance, hexagons *and* trapezoids are blue) or increase the complexity by having a full mixture of colors (such as blue hexagon, green trapezoid, red hexagon, green trapezoid, yellow hexagon, red trapezoid). In this way,

Figure 2: Bug pattern

children are encouraged to problem solve and focus on the repeating attribute without relying on color.

While young learners can use shape, size, and color as attributes for patterns, they also can use numerous other attributes. Simple position patterns such as “up, down, up, down . . .” or “top, side, front, top, side, front . . .” encourage children to use spatial orientations as the repeating features. Virtually any sorting category—the number of holes in buttons, the texture of shells—can be used as a repeating attribute. Kindergartner Aneil creates a repeating pattern with plastic bugs from the science center (see Figure 2). When I ask Aneil to read his pattern for the class, he proudly exclaims, “Wings, no wings, wings, no wings, wings, no wings . . .” His algebraic thinking and creativity are simply brilliant! He is isolating one attribute but also negating several others, which is higher level thinking in action.

When working with repeating patterns, it is important to present the repeating nature of the pattern. A pattern is not a pattern until it repeats. For example, we should not present children with and say continue the pattern, because there is not yet a pattern in this series. However, by displaying the repetitive nature, as in the pattern is apparent and can be extended.

Not only do young children need many experiences with repeating patterns, they also can work with growing patterns that increase (or decrease) by a constant amount. The simplest of all growing patterns increases by one and begins with a small number. For example, “1, 2, 3, 4, 5, 6...” is a growing pattern based on a constant change of plus one. This is best understood by young children through concrete representation. With linking cubes, the plus-one pattern can be visually shared with children. Stack linking cubes in towers and line up the towers in order of size. Children are quick to notice that the result looks like steps or a staircase. They can explain what should come next in the pattern by making the next tower.

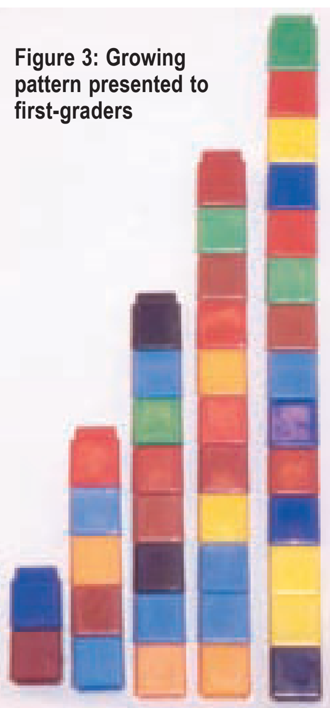


Figure 3: Growing pattern presented to first-graders

To increase the level of difficulty, we can present children with growing patterns that increase (or decrease) by more than one and start with numbers/quantities other than one or zero. Figure 3 shows a growing pattern presented to first-graders. Initially the children share what they discern about the blocks. Ebony and

"This is a growing pattern. A growing pattern goes like a stairs—you can add for 2, 3, 4, or 5, and it grows and grows"

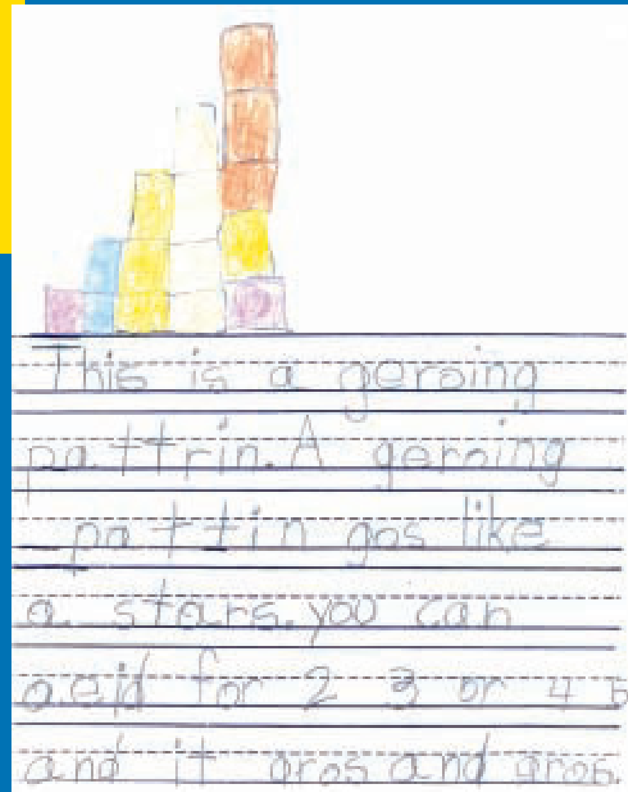


Figure 4: Lex’s pattern

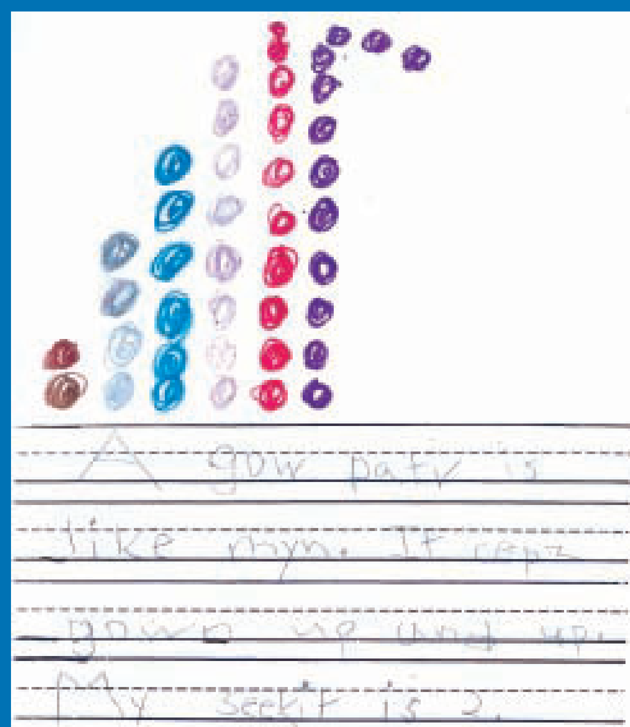


Figure 5: Khavin’s growing pattern

Christopher notice the blocks are lined up. Juan explains, "It looks like steps to a giant's house—'cause the steps are really big." Juan's comment opens the door for the teacher to ask, "How big are the steps?" Maria quickly responds, "Each one is bigger by the same, and that's 3—so the next one is 14!" The class agrees, and Maria constructs a stack of 14 cubes. The lesson continues and the children construct stacks of 17, 20, 23, and 26. At this point, Khavin exclaims, "This could go on forever!" "You're right, Khavin," says the teacher, "and that's why mathematicians use three dots to show that a series continues!"

Over the next several days the children use their newly acquired algebraic thinking skills to construct their own growing patterns in their math journals. Lex has a solid understanding of simple growing patterns. She writes, "This is a growing pattern. A growing pattern goes like a stairs—you can add for 2, 3, 4, or 5, and it grows and grows" (see Figure 4). Khavin, who depicts a growing pattern that increases by two, writes, "A growing pattern is like mine. It keeps growing up and up. My secret is 2" (see Figure 5).

If children experience and begin to understand equality in the early grades, they can readily solve for unknown variables. . . . By asking how many are needed to make the quantities the *same* or the situation *fair*, we incorporate the concept of equality.

The children's illustrations and words give the teacher a clear understanding of where each child is in her or his construction of knowledge. She sees that Lex is ready to try patterns that increase (or decrease) by more than one; and Khavin, who understands repeated function through the idea of a "secret," is ready for more elaborate patterns. Each child needs the appropriate challenges to continue to make meaningful connections within these algebraic expressions.

Growing patterns and repeating patterns are often more distinct when displayed in a linear fashion. However, some real-world patterns are nonlinear. Concentric patterns that grow from the inside to the outside are one type of nonlinear pattern and can be found in artwork, fabric designs, and nature. The nautilus shell provides a mathematically brilliant concentric pattern (the ratio of the chambers within the spiral is a perfect 1 to 1.6180). Other nonlinear patterns are common in floor tiles, buildings, even windblown sand. We need to encourage young children to notice and describe the many types of patterns found in their worlds.

Figure 6: Creating balance

Mathematical situations and structures

Representing and analyzing mathematical situations and structures is a major component of algebraic thinking. To build concepts for later work with properties such as commutativity ($a + b = b + a$), associativity ($[a + b] + c = a + [b + c]$), and equivalent forms ($a + b = c$), children need many experiences with mathematical situations and structures through representations and analyses of equality. As noted in the math standards for prekindergarten through second grade, "Equality is an important algebraic concept that students must encounter and begin to understand in the lower grades" (NCTM 2000, 94). If children experience and begin to understand equality in the early grades, they can readily solve for unknown variables.

With this in mind, we need to offer young children many experiences with recognizing, defining, creating, and maintaining equality. Pan balance scales are one way to demonstrate equality. Young children need to use scales and make scales. They need to talk about *equal/not equal, same/different, more/less,* and *balanced/unbalanced.* It is through authentic dialogue that children construct meaning related to the concept of algebraic equality. Children can manipulate the scales with everyday objects to show equality through the idea of balance. Prekindergartner Joey explains, "You know it's balanced when it's really straight." "Yeah," adds Robert, "it's not going to one side—and that's what balance is all about" (see Figure 6). In this learning situation the children recognize and define equality through the idea of balance. They are ready for experiences with creating and maintaining equality (see "First-Graders Discover Concept of Balance in a Simple Experiment," pp. 18–19).

First-Graders Discover Concept of Balance in a Simple Experiment

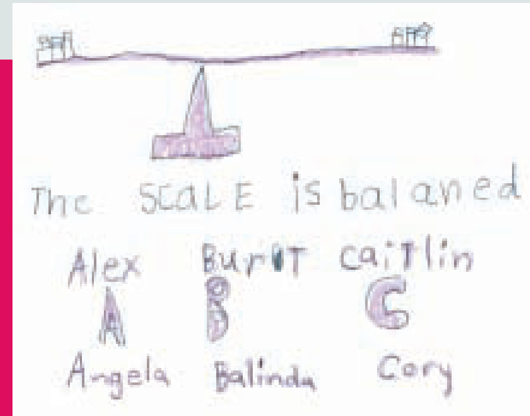


Figure 7: Ebony's illustration and description

By using real object representations, young children are capable of masterful algebraic thinking. The first-graders here are using simple scales and six film canisters with varying amounts of sand inside. Two canisters are filled completely (A=Alex and A=Angela); two are filled halfway (B=Burt and B=Balinda); and two have only a few grains of sand (C=Cory and C=Caitlin).

After several experiences with comparing and ordering the film canisters by weight, the children are invited to participate in a performance task related to the notion of algebraic equality. The teacher sets up the performance task with the following vignette.

> There are six children in the Can family (each represented by a film canister). At the park, all six children want to ride the super seesaw at the same time. To help the Can family decide who should ride together, place the Cans on the scale to show balance. Draw a picture of what you find out.

The children discover that balance must be created by placing the cans on the scale in the following equation: $A + C + B = C + A + B$. Then the teacher asks, "What if Alex wants to get off the seesaw?"

Josh describes the needed sequence of events: "If Alex wants to get off the seesaw, then Angela has to get off too, because she is the one who weighs the same." In first-grade language, Josh describes the algebraic notion of maintaining equality within an expression (that is, $B + C - A = C + B - A$). Without being prompted by further questions, Pfeifer adds, "And if Caitlin wants to get off the seesaw, Cory has to get off because they are the same." Many other children see the connections and note that Burt and Balinda are also the "same" and would have to be removed together to keep the balance or maintain equality.

In this activity, young children can see in concrete terms what maintaining balanced equations encompasses. Ebony includes a precise illustration and declares, "The scale is balanced" (see Figure 7). Josh writes, "We made it balance by putting three

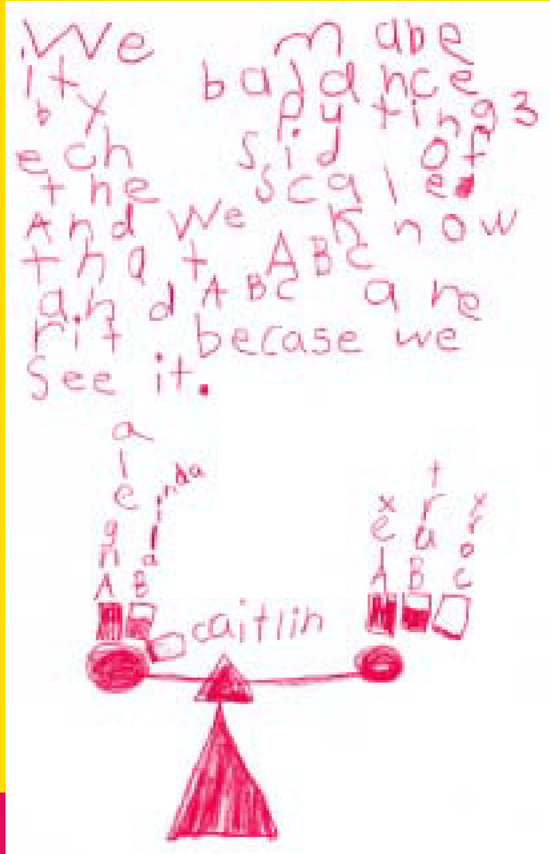


Figure 8: Josh's illustration and description

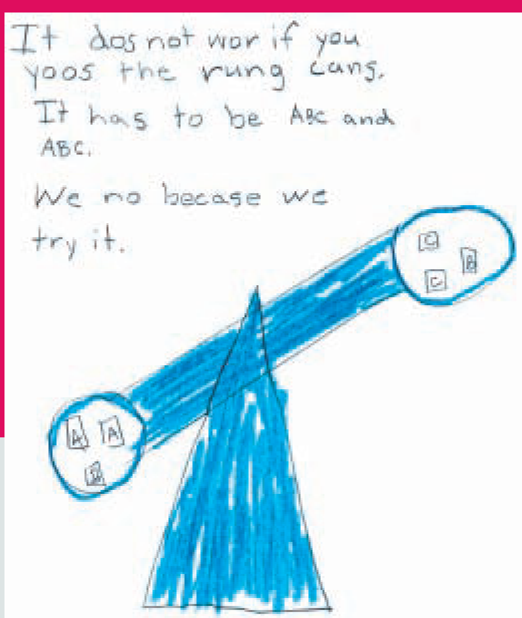


Figure 9: Pfeifer's illustration and description

on each side of the scale. And we know that ABC and ABC are right because we see it" (see Figure 8). Josh's illustration impressively includes the amount of sand in each canister. Pfeifer explains, "It does not work if you use the wrong cans. It has to be ABC and ABC. We know because we tried it" (see Figure 9). Her illustration depicts this important part of the learning process.

Many other mathematical situations emphasize equality. We know that children often announce, "She has more than me!" or "I don't have enough—it's not fair." Instead of concentrating only on the social implications of these statements, we can focus on the mathematics. By asking how many are needed to make the quantities the *same* or the situation *fair,* we incorporate the concept of equality, making algebraic thinking part of everyday life.

Additionally, children can engage in a variety of tasks involving mathematics situations and structures with an assortment of manipulatives. In a classroom rich with algebraic thinking in action, children link cubes to make towers that are equal in quantity and thus equal in height. Others string beads and discuss why one string is longer than the other. One kindergartner says to a classmate, "Ours are not the same. Mine is longer because I have two more beads than you." The children also use the arrangement of ten-frames to show equality by means of different arrangements—one shows six red counters and four yellow counters while another shows five red counters and five yellow counters. The children explain that both frames of 10 show the same value. "See," says Justin, "they are both 10. It doesn't matter that mine has more red than his." Such opportunities with mathematical situations and structures offer the experiences young children need to build strong foundations in algebraic thinking.

Models of quantitative relationships

Models of quantitative relationships also require hands-on approaches to build awareness and understanding of values. Ask young children a question such as "How many eyes in a group of four people?" and you may witness a variety of problem-solving approaches. Some children may gather together in a group of four and count the number of eyes. Some may draw pictures of people with eyes. Still others may use four paper plates with two counters on each plate. Some may be ready to write 2 + 2 + 2 + 2 = 8. As educators, we encourage children's thinking by asking questions such as "How do you know there are eight eyes?" "What other ways can you show this situation?" "What kind of pattern do you see?" "What if there were only three people?" These questions open the doors for dialogue that prompts further thinking about models of quantitative relationships.

"What other ways can you show this situation?" "What kind of pattern do you see?" "What if there were only three people?"

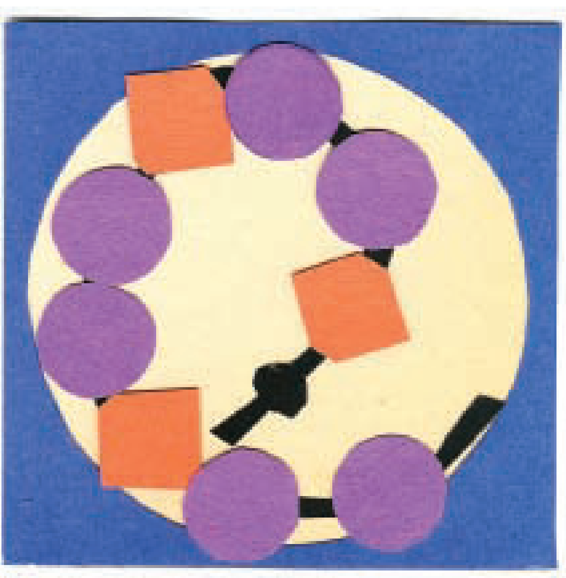
Teachers can use models to represent and teach quantitative relationships during everyday classroom experiences. For example, in Mr. Carson's prekinder-garten classroom, children model and discuss quantitative relationships during snack time. Mr. Carson invites four children to each choose three snacks from a plate of crackers and pretzels. Elena chooses three crackers, Tyrone picks one pretzel and two crackers, Martin selects two pretzels and one cracker, and Rosa decides to have three pretzels. The teacher encourages the children to compare and contrast their snack choices. He asks them if they have the same amount. Martin answers, "Mine is more pretzels, but I know I have 3." Rosa replies, "I have all pretzels—my 3 is easy." Tyrone adds, "Threes can be different." In this exchange the children represent and verbalize an understanding of the different ways to compose the quantity of three. In so doing, they explore models of quantitative relationships in a real-life context.

The use of exact amounts differentiates quantitative change from qualitative change.

Change

The final big idea in algebraic thinking is change. As noted in the math standards, "The understanding that most things change over time, that such changes can be described mathematically, and that changes are predictable helps lay a foundation for applying mathematics" (NCTM 2000, 95). We must encourage young children to notice and describe many different changes.

According to the standards, there are two algebraically oriented types of change: qualitative and quantitative. *Qualitative change* entails many familiar experiences that are part of children's lives: A pair of shoes feels smaller as the child's feet grow; the sunflower is taller than it was last week; a bucket fills with water as the rain continues all day. All of these changes are qualitative. They are described with relative mathematical labels such as *smaller, taller,* and *fuller.* The changes occur over time and are fairly predictable. *Quantitative changes* are also part of children's lives: The child's shoe size changes from 10 to 11; the sunflower grows 3 centimeters in one week; the amount of water in the bucket increases by 50 milliliters every 30 minutes of a three-hour rainfall. The mathematic language to describe changes incorporates more precise numeric language. Essentially, the use of exact amounts differentiates quantitative change from qualitative change. Young children need to notice and think about both types of change.



Play offers young children many experiences with change. While filling a bag with blocks, prekindergartners notice that the bag gets heavier with each additional block. Using a cardboard ramp and a tennis ball, kindergartners discover that changing the height of the ramp changes the distance the ball rolls. With chain links and weights, first-graders discover and discuss how the length of the chain influences the number of pendulum-type swings. In each of these situations, the children analyze changes that occur as they experiment while playing.

Many songs and finger plays are built around repeating patterns and growing patterns.

Conclusion

As teachers, our role is to facilitate children's learning through play. As expressed in the joint position statement, we must "provide ample time, materials, and teacher support for children to engage in play, a context in which they explore and manipulate mathematical ideas with keen interest" (NAEYC & NCTM 2002, 10). As with the concept of change, children can construct knowledge about each of the major components of algebra through play. They discover patterns in clothing, calendars, and pictures and as they participate in classroom routines. They see mathematical situations and structures as they compare the amounts of juice in each other's cup and the number of stickers by each classmate's name on a graph about family size. Furthermore, young children create models of quantitative relationships as they make change from the cash register and help the teacher set out paint containers at the easels. They also find algebra in stories, songs, poems, and finger plays. Many songs and finger plays are built around repeating patterns and growing patterns. Numerous stories are constructed around predictable change, mathematical situations and structures,

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and models of quantitative relationships. The opportunities for algebraic thinking are nearly endless.

If early childhood educators are to enhance children's outcomes, encouraging algebraic thinking in the early years is essential. We can begin by offering many experiences with varied types of patterns, mathematical situations and structures, models of quantitative relationships, and change. We can enhance young children's learning by giving appropriate challenges that incrementally increase the level of complexity and by asking questions that promote mathematical dialogue. To open future gates and remove potential barriers to academic pursuits, we must build the necessary foundations for young children by not only incorporating *more* algebraic thinking experiences but also by requiring that these experiences are of *high quality*. In so doing, we will better prepare young children for the opportunities that await them!

References

Copley, J.V. 2000. *The young child and mathematics.* Washington, DC: NAEYC.

Lott, J.W., ed. 2000. Algebra? A gate? A barrier? A mystery! *Mathematics Education Dialogues* 3 (2): 1–12.

Moses, R.P. 2001. Algebra and activism: Removing the shackles of low expectations—A conversation with Robert P. Moses. *Educational Leadership* 59 (2): 6–11.

NAEYC & NCTM (National Council of Teachers of Mathematics). 2002. Early childhood mathematics: Promoting good beginnings. Joint position statement. Washington, DC: NAEYC and Reston, VA: NCTM. Online: www. naeyc.org/resources/position-statements/psmath.htm.

NCTM (National Council of Teachers of Mathematics). 2000. *Principles and standards for school mathematics.* Reston, VA: NCTM.